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SEPARATE CARRY STORAGE ADDERS

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Introduction

The following considerations will be applicable to the pseudo-adder of any base with carry storage, but for simplicity let us confine our discussion to the base 4 adder, which will be the heart of the arithmetic unit of the New Illinois Computer.

Let a_{2n+1}, a_{2n}, a_{2n-1}, ...be the previous content of the accumulator with the carry storage digits b_{2n}, b_{2n-2},...; let x_{2n+1}, x_{2n}, x_{2n-1},...be the operand, and a'_{2n+1}, a'_{2n}, a'_{2n-1},...be the result with the new carry storage digits, b'_{2n}, b'_{2n-2}, ..., where the index increases with decreasing significance of the digital position. (c. f. Figure 1)

b_{2n-2}		b_{2n}	
a_{2n-2}	a_{2n-1}	a_{2n}	a_{2n+1}
x_{2n-2}	x_{2n-1}	x_{2n}	x_{2n+1}
a'_{2n-2}	a'_{2n-1}	a'_{2n}	a'_{2n+1}
b'_{2n-2}		b'_{2n}	

Figure 1

The variety in such an adder structure arises from the fact that there are various schemes concerning what kind of carry we store as b'_{2n}, b'_{2n-2},..., and what kind of carry we allow to propagate to more significant digital positions. This concept has not been explicitly discussed before.

General Consideration

Let the carry digit to the (2n-1)-th digital position be u_{2n-1}. Then the logical maximum we get as the carry to the (2n-2)-th digital position will be

$$c'_{2n-2} = u_{2n-1} (a_{2n-1} \oplus x_{2n-1}) \vee a_{2n-1} x_{2n-1}. \quad (1)$$

The new stored carry digit b'_{2n-2} is a part of c'_{2n-2}, and the remaining portion p'_{2n-2} must be propagated to the next more significant digital position. Between b'_{2n-2} and p'_{2n-2} must be the following simple relation:

$$b'_{2n-2} \vee p'_{2n-2} = c'_{2n-2}, \tag{2}$$

$$b'_{2n-2} \cdot p'_{2n-2} = 0. \tag{3}$$

Since, with an arbitrary logical function A_{2n-2},

$$b'_{2n-2} = c'_{2n-2} \cdot A_{2n-2} \tag{4}$$

and

$$p'_{2n-2} = c'_{2n-2} \cdot \overline{A}_{2n-2} \tag{5}$$

always satisfy both (2) and (3), we can design infinitely many kinds of adders corresponding to the infinite number of arbitrary logical functions, though only some of them will be worth investigating from the practical point of view.

Before discussing this separation of carry in detail, however, we need some general considerations. Let us formulate the result, a'_{2n}, a'_{2n-1}, and c'_{2n-2} in terms of the given digits and p'_{2n}, which is the carry that will be propagated from the previous lower significant stage.

At the beginning we hope to have an important relation,

$$b_{2n} \cdot p'_{2n} \equiv 0, \tag{6}$$

which is, as will be shown later on, valid with some limitations. With this assumption, the logical quantity

$$t_{2n} = b_{2n} \vee p'_{2n} \tag{7}$$

can take care both of b_{2n} and of p'_{2n}. Then we have

$$a'_{2n} = t_{2n} \oplus a_{2n} \oplus x_{2n} \tag{8}$$

$$u_{2n-1} = t_{2n} (a_{2n} \oplus x_{2n}) \vee a_{2n} x_{2n} \tag{9}$$

$$a'_{2n-1} = u_{2n-1} \oplus a_{2n-1} \oplus x_{2n-1} \tag{10}$$

and

$$c'_{2n-2} = u_{2n-1} (a_{2n-1} \oplus x_{2n-1}) \vee a_{2n-1} x_{2n-1}. \tag{11}$$

Introducing an arbitrary logical function A_{2n-2}, we can separate the carry c'_{2n-2} into b'_{2n-2} and p'_{2n-2} theoretically in an infinite number of ways. however, from the practical point of view, there will be the following limitations:

(i) We hope to keep the relation

$$b'_{2n-2} \cdot p''_{2n-2} \equiv 0 \tag{12}$$

where p''_{2n-2} is the carry that will be propagated to the next more significant stage in the next addition. This is the same relation as (6) which we have assumed, and if we cannot take advantage of this relation, we have the possibility of the double carries of b'_{2n-2} and p''_{2n-2} at the same time which calls for a much more complicated adder structure which we should avoid.

(ii) The equation (11) is rewritten in terms of p'_{2n}, b_{2n}, a_{2n}, x_{2n}, a_{2n-1}, and x_{2n-1} as

$$\begin{aligned} c'_{2n-2} = {} & p'_{2n} \cdot (a_{2n} \oplus x_{2n}) \cdot (a_{2n-1} \oplus x_{2n-1}) \\ & \vee b_{2n} \cdot (a_{2n} \oplus x_{2n}) \cdot (a_{2n-1} \oplus x_{2n-1}) \\ & \vee a_{2n} \cdot x_{2n} \cdot (a_{2n-1} \oplus x_{2n-1}) \vee a_{2n-1} \cdot x_{2n-1}. \end{aligned} \tag{13}$$

We do not want to introduce any more logical variables, though it is possible to separate the third term, for instance, as

$$a_{2n} \cdot x_{2n} \cdot (a_{2n-1} \oplus x_{2n-1}) \cdot a_{2n+1} \vee a_{2n} \cdot x_{2n} \cdot (a_{2n-1} \oplus x_{2n-1}) \cdot \bar{a}_{2n+1},$$

introducing a new variable a_{2n+1}, because this only increases the complexity of the adder construction. For the same reason, we do not intend either to separate the term $a_{2n-1} \cdot x_{2n-1}$ as

$$a_{2n} \cdot a_{2n-1} x_{2n-1} \vee \bar{a}_{2n} \cdot a_{2n-1} \cdot x_{2n-1},$$

for instance. However, we may separate, for instance, the term

$$a_{2n} \cdot x_{2n} \cdot (a_{2n-1} \oplus x_{2n-1})$$

into

$$a_{2n} \cdot x_{2n} \cdot a_{2n-1} \cdot \bar{x}_{2n-1} \vee a_{2n} \cdot x_{2n} \cdot \bar{a}_{2n-1} \cdot x_{2n-1},$$

since we hope to employ diode logic as far as we can.

(iii) The carry corresponding to the first term of (13) ought to be stored, since this is the carry propagated from the previous lower significant stage. Otherwise, a propagation of a carry in a long chain will take place, and the carry storage scheme will be no longer useful.

Validity of the Relation $b_{2n} \cdot p'_{2n} \equiv 0$

Since we have assumed the relation,

$$b_{2n} \cdot p'_{2n} \equiv 0, \tag{6}$$

we must examine its validity. We can try this by mathematical induction, because $b_{2n} = 0$ and hence (6) is valid before the first addition after carry assimilation, and because (6) is always valid for the least significant stage, since there is no carry storage and no carry propagation.

Suppose (6) was valid at the n-th stage for the m-th addition. Then all the relations from (7) to (10) are correct. Then it is sufficient enough for us to examine the validity of

$$b'_{2n-2} \cdot p''_{2n-2} = 0 \tag{12}$$

at the (n-1)-th stage for the (m+1)-th addition. By examining this, we can also clarify the limitation on the carry separation stated in (i).

In the following discussion, let us omit 2n from all the indices of variables for simplicity; for instance let a_{2n-1} be written a_{-1}. Let

$$\begin{aligned} f_0 &= ax \\ f_1 &= a\bar{x} \\ f_2 &= \bar{a}x \\ f_3 &= \overline{ax}, \end{aligned} \tag{14}$$

and

$$\begin{aligned} k_0 &= a_{-1} \cdot x_{-1} \\ k_1 &= a_{-1} \cdot \bar{x}_{-1} \\ k_2 &= \bar{a}_{-1} \cdot x_{-1} \\ k_3 &= \bar{a}_{-1} \cdot \bar{x}_{-1} \end{aligned} \tag{15}$$

It is clear that all $f_i \cdot f_j$, $(i \neq j)$, and $k_i \cdot k_j$, $(i \neq j)$, are void.

With the use of (14) and (15), we obtain

$$a' = tf_0 \vee \bar{t}f_1 \vee \bar{t}f_2 \vee tf_3 \tag{16}$$

$$\bar{a}' = \bar{t}f_0 \vee tf_1 \vee tf_2 \vee \bar{t}f_3$$

$$u_{-1} = f_0 \vee tf_1 \vee tf_2$$

$$\bar{u}_{-1} = \bar{t}f_1 \vee \bar{t}f_2 \vee f_3$$

$$\begin{aligned} a'_{-1} &= u_{-1}\cdot k_0 \vee \bar{u}_{-1}\cdot k_1 \vee \bar{u}_{-1}\cdot k_2 \vee u_{-1}\cdot k_3 \\ &= f_0\cdot k_0 \vee f_0\cdot k_3 \vee tf_1\cdot k_0 \vee \bar{t}f_1\cdot k_1 \vee \bar{t}f_1\cdot k_2 \vee tf_1\cdot k_3 \\ &\quad \vee tf_2\cdot k_0 \vee \bar{t}f_2\cdot k_1 \vee \bar{t}f_2\cdot k_2 \vee tf_2\cdot k_3 \vee f_3\cdot k_1 \vee f_3\cdot k_2 \end{aligned} \tag{17}$$

$$\begin{aligned} \bar{a}'_{-1} &= \bar{u}_{-1}\cdot k_0 \vee u_{-1}\cdot k_1 \quad u_{-1}\cdot k_2 \vee \bar{u}_{-1}\cdot k_3 \\ &= f_0\cdot k_1 \vee f_0\cdot k_2 \vee \bar{t}f_1\cdot k_0 \vee tf_1\cdot k_1 \vee tf_1\cdot k_2 \vee \bar{t}f_1\cdot k_3 \\ &\quad \vee tf_2\cdot k_0 \vee tf_2\cdot k_1 \vee tf_2\cdot k_2 \vee \bar{t}f_2\cdot k_3 \vee f_3\cdot k_0 \vee f_3\cdot k_3 \end{aligned}$$

$$\begin{aligned} c'_{-2} &= k_0 \vee f_0\cdot k_1 \vee f_0\cdot k_2 \vee b(f_1\cdot k_1 \vee f_1\cdot k_2 \vee f_2\cdot k_1 \vee f_2\cdot k_2) \\ &\quad \vee p'(f_1\cdot k_1 \vee f_1\cdot k_2 \vee f_2\cdot k_1 \vee f_2\cdot k_2). \end{aligned}$$

Taking into account the limitation (iii) stated in the previous section, we have

$$\begin{aligned} b'_{-2} &= B_0\cdot k_0 \vee B_{01}\cdot f_0\cdot k_1 \vee B_{02}\cdot f_0\cdot k_2 \vee B_{11}\cdot bf_1\cdot k_1 \vee B_{12}\cdot bf_1\cdot k_2 \\ &\quad \vee B_{21}\cdot bf_2\cdot k_1 \vee B_{22}\cdot bf_2\cdot k_2 \vee p'[f_1\cdot k_1 \vee f_1\cdot k_2 \vee f_2\cdot k_1 \vee f_2\cdot k_2] \end{aligned} \tag{18}$$

and

$$\begin{aligned} p'_{-2} &= \bar{B}_0\cdot k_0 \vee \bar{B}_{01}\cdot f_0\cdot k_1 \vee \bar{B}_{02}\cdot f_0\cdot k_2 \vee \bar{B}_{11}\cdot bf_1\cdot k_1 \vee \bar{B}_{12}\cdot bf_1\cdot k_2 \\ &\quad \vee \bar{B}_{21}\cdot bf_2\cdot k_1 \vee \bar{B}_{22}\cdot bf_2\cdot k_2, \end{aligned}$$

where B_0, $B_{01}, \ldots\ldots B_{22}$ are arbitrary coefficients, each of which is either 1 or 0.

Since

$$p''_{-2} = \overline{B}_0 \, k'_0 \vee \overline{B}_{01} \, f'_0 \, k_1 \vee \overline{B}_{02} \, f'_0 \, k'_2 \vee \overline{B}_{11} \, b'f'_1 \, k'_1 \vee \overline{B}_{12} \, b'f'_1 \, k'_2$$

$$\vee \; \overline{B}_{21} \, b'f'_2 \, k'_1 \vee \overline{B}_{22} \, b'f'_2 \, k'_2,$$

where f'_0, f'_1,.....k'_0 .....mean $a' \cdot x'$, $a' \cdot \overline{x}'$, . . . , $a'_{-1} \cdot x'_{-1}$, . . . , respectively, we can now represent $b'_{-2} \; p''_{-2}$ explicitly as follows:

$$b'_{-2} \, p''_{-2} = B_0 \, k_0 \left\{ x' \, x'_{-1} \, [\overline{B}_{01} \, tf_0 \vee \overline{B}_{02} \, (\overline{t}f_1 \vee \overline{t}f_2 \vee tf_3)] \right.$$

$$\vee \; b'[\overline{B}_{11} \, \overline{x}' \, \overline{x}'_{-1} \, tf_0 \vee \overline{B}_{12} \, \overline{x}' \, x'_{-1}(\overline{t}f_1 \vee \overline{t}f_2 \vee tf_3)$$

$$\left. \vee \; \overline{B}_{21} \, x' \, \overline{x}'_{-1}(\overline{t}f_0 \vee tf_1 \vee tf_2) \vee \overline{B}_{22} \, x' \, x'_{-1} \, \overline{t}f_3] \right\}$$

$$\vee \; \overline{B}_{22} \, x' \, x'_{-1} \, b'[\overline{t}f_0 \, (B_{01} \, k_1 \vee B_{02} \, k_2) \vee$$

$$bt(B_{11} \, f_1 \, k_1 \vee B_{12} \, f_1 \, k_2 \vee B_{21} \, f_2 \, k_1)]$$

$$\vee \; \overline{B}_{12} \, \overline{x}' \, x'_{-1} \, b' \, tf_0 \, (B_{01} \, k_1 \vee B_{02} \, k_2)$$

$$\vee \; B_{01} \, \overline{B}_{02} \, x' \, x'_{-1} \, tf_0 \, k_1 \, . \qquad (19)$$

In this deduction, it should be noted that

$$b' \, p' = 0 \qquad (3')$$

$$p' \, \overline{t} = 0$$
$$b \, \overline{t} = 0. \qquad (7')$$

(7') is obvious from (7), and (3') is due to the definition. By the way, it is also noted that (3') is an entirely different relation from (6) which we have assumed, and also from (12) which we are examing.

We can easily see that each term of (19) cannot always be void unless the coefficient is zero. Therefore for the validity of (12) we must have

$$B_0 \; (\overline{B}_{01} \vee \overline{B}_{02} \vee \overline{B}_{11} \vee \overline{B}_{12} \vee \overline{B}_{21} \vee \overline{B}_{22})$$

$$\vee \; \overline{B}_{22} \; (B_{01} \vee B_{02} \vee B_{11} \vee B_{12} \vee B_{21})$$

$$\vee \; B_{01} \, \overline{B}_{12} \vee B_{01} \, \overline{B}_{02} \vee B_{02} \, \overline{B}_{12} \equiv 0. \qquad (20)$$

(12) is valid under this condition which is also an explicit expression of the limitation (i).

Inspecting the condition (20), it can be easily shown that $B_0 = 1$ is a special case in which coefficients B_{01}, B_{02}, B_{11}, B_{12}, B_{21} and B_{22} must be also 1, and $B_{22} = 0$ is another special case in which all other coefficients, B_0, B_{01}, B_{02}, B_{11}, B_{12} and B_{21} must be also zero.

Excluding these two special cases,

$$B_0 = \bar{B}_{22} \equiv 0, \tag{21}$$

and (20) is read as

$$B_{01} \bar{B}_{12} \vee B_{01} \bar{B}_{02} \vee B_{02} \bar{B}_{12} \equiv 0, \tag{22}$$

which calls for the validity of one of the following three conditions:

$$\left.\begin{aligned} B_{01} &= B_{02} \equiv 0 \\ B_{01} &= \bar{B}_{12} \equiv 0 \\ \bar{B}_{12} &= \bar{B}_{02} \equiv 0. \end{aligned}\right\} \tag{23}$$

Since there are five coefficients besides B_0 and B_{22}, and two of them are always bound by (23), we shall have 18 cases as shown in Table 1, including the two special cases mentioned above.

It is also worth while to note that there are simple relations among the new stored carry b'_{-2} and the sum digits a'_{-1} and a'. From (17) and (18), we have

$$\begin{aligned} b'_{-2}\, a'_{-1} &= B_0\, k_0\, (f_0 \vee tf_1 \vee tf_2) \\ &= B_0\, a_{-1}\, x_{-1}\, [t\, (a \oplus x) \vee a\, x] \end{aligned} \tag{24}$$

Except for the very special case in which $B_0 = 1$, this is always zero. From (16) and (18), we also have

$$\begin{aligned} b'_{-2}\, a' &= B_0\, k_0\, (tf_0 \vee \bar{t}f_1 \vee \bar{t}f_2 \vee tf_3) \\ &\quad tf_0\, (B_{01}\, k_1 \vee B_{02}\, k_2) \end{aligned}$$

Excluding the case $B_0 = 1$, this is read as

$$b'_{-2}\, a' = t\, a\, x\, (B_{01}\, a_{-1}\, \bar{x}_{-1} \vee B_{02}\, \bar{a}_{-1}\, x_{-1}), \tag{25}$$

and with the condition (23), we shall have the following three cases:

$$b'_{-2}\, a' = 0 \qquad (B_{01} = B_{02} \equiv 0)$$

$$= t\, a\, x\, \bar{a}_{-1}\, x_{-1} \qquad (B_{01} = \bar{B}_{12} = \bar{B}_{02} \equiv 0)$$

$$= t\, a\, x\, (a_{-1} \oplus x_{-1}). \qquad (\bar{B}_{01} = \bar{B}_{12} = \bar{B}_{02} \equiv 0)$$

The value of $b'_{-2}\, a'$ is also indicated in Table 1. **)

It should be noted that the structure of the adder is not necessarily recursive, provided that the structure of each stage is such that the relation (6) is valid. Therefore it is even possible to change the structure from stage to stage, picking up an arbitrary one from the table. From a practical viewpoint, especially from that of maintenance, however, it is highly desirable to have a recursive structure.

With the use of this table and equations from (7) to (10), we can immediately write down the Boolean equation for each structure of the base 4 pseudo-adder.

Some Considerations

The general tendency seen from the table is that the circuit to generate p'_{-2} becomes simpler as we go down through the table and on the contrary the circuit to generate b'_{-2} becomes more complicated, and also that on the whole, the structures in the lower part of the table seem to be simpler than those in the upper rows.

The structure numbered 17 has been considered for the new Illinois computer. The structure 18, which seems to be the simplest, has been abandoned due to the lack of the relation,

$$b'_{-2}\, a'_{-1} = 0. \qquad (26)$$

However, the structures numbered from 1 to 9, for which another relation,

$$b'_{-2}\, a' = 0 \qquad (27)$$

also holds besides (26), will be worth our while to examine more in detail,

**) The ** footnote in Table 1 refers to line 18 of that Table.

Table 1

All of the Practically Possible Varieties of Base 4 Pseudo-adder Constructions

#	B_0	B_{01}	B_{02}	B_{11}	B_{12}	B_{21}	B_{22}	b'_{-2}	p'_{-2}	$b'_{-2}a'$
1	0	0	0	0	0	0	0	$p'(a \oplus x)(a_{-1} \oplus x_{-1}) \equiv (b'_{-2})_0$	$b(a \oplus x)(a_{-1} \oplus x_{-1}) \vee ax(a_{-1} \oplus x_{-1}) \vee a_{-1}x_{-1}$	
2	0	0	0	0	0	0	1	$(b'_{-2})_0 \vee b\bar{a}x\bar{a}_{-1}x_{-1}$	$b(a \oplus x)a_{-1}x_{-1} \vee bax\bar{a}_{-1}x_{-1} \vee ax(a_{-1} \oplus x_{-1}) \vee a_{-1}x_{-1}$	
3	0	0	0	0	0	1	1	$(b'_{-2})_0 \vee b\bar{a}x(a_{-1} \oplus x_{-1})$	$ba\bar{x}(a_{-1} \oplus x_{-1}) \vee ax(a_{-1} \oplus x_{-1}) \vee a_{-1}x_{-1}$	
4	0	0	0	0	1	0	1	$(b'_{-2})_0 \vee b(a \oplus x)\bar{a}_{-1}x_{-1}$	$b(a \oplus x)a_{-1}\bar{x}_{-1} \vee ax(a_{-1} \oplus x_{-1}) \vee a_{-1}x_{-1}$	
5	0	0	0	0	1	1	1	$(b'_{-2})_0 \vee b(a \oplus x)\bar{a}_{-1}x_{-1} \vee b\bar{a}xa_{-1}\bar{x}_{-1}$	$b\bar{a}x\bar{a}_{-1}x_{-1} \vee ax(a_{-1} \oplus x_{-1}) \vee a_{-1}x_{-1}$	0
6	0	0	0	1	0	0	1	$(b'_{-2})_0 \vee ba\bar{x}a_{-1}\bar{x}_{-1} \vee b\bar{a}x\bar{a}_{-1}x_{-1}$	$ba\bar{x}\bar{a}_{-1}x_{-1} \vee b\bar{a}xa_{-1}\bar{x}_{-1} \vee ax(a_{-1} \oplus x_{-1}) \vee a_{-1}x_{-1}$	
7	0	0	0	1	0	1	1	$(b'_{-2})_0 \vee b\bar{a}x(a_{-1} \oplus x_{-1})\ ba\bar{x}a_{-1}\bar{x}_{-1}$	$ba\bar{x}\bar{a}_{-1}x_{-1} \vee ax(a_{-1} \oplus x_{-1}) \vee a_{-1}x_{-1}$	
8	0	0	0	1	1	0	1	$(b'_{-2})_0 \vee b(a \oplus x)\bar{a}_{-1}x_{-1} \vee ba\bar{x}a_{-1}\bar{x}_{-1}$	$b\bar{a}xa_{-1}\bar{x}_{-1} \vee ax(a_{-1} \oplus x_{-1}) \vee a_{-1}x_{-1}$	
9	0	0	0	1	1	1	1	$(b'_{-2})_0 \vee b(a \oplus x)(a_{-1} \oplus x_{-1})$	$ax(a_{-1} \oplus x_{-1}) \vee a_{-1}x_{-1}$	
*	0	0	1	0	0	0	1			
*	0	0	1	0	0	1	1			
10	0	0	1	0	1	0	1	$(b'_{-2})_0 \vee b(a \oplus x)\bar{a}_{-1}x_{-1} \vee ax\bar{a}_{-1}x_{-1}$	$b(a \oplus x)a_{-1}\bar{x}_{-1} \vee axa_{-1}\bar{x}_{-1} \vee a_{-1}x_{-1}$	
11	0	0	1	0	1	1	1	$(b'_{-2})_0 \vee b(a \oplus x)\bar{a}_{-1}x_{-1} \vee b\bar{a}xa_{-1}\bar{x}_{-1} \vee ax\bar{a}\ x_{-1}$	$ba\bar{x}a_{-1}\bar{x}_{-1} \vee ax\ a\ \bar{x}_{-1} \vee a_{-1}x_{-1}$	
*	0	0	1	1	0	0	1			$tax\,a_{-1}x_{-1}$
*	0	0	1	1	0	1	1			
12	0	0	1	1	1	0	1	$(b'_{-2})_0 \vee b(a \oplus x)\bar{a}_{-1}x_{-1} \vee ba\bar{x}a_{-1}\bar{x}_{-1} \vee ax\bar{a}_{-1}x_{-1}$	$b\bar{a}xa_{-1}\bar{x}_{-1} \vee axa_{-1}\bar{x}_{-1} \vee a_{-1}x_{-1}$	
13	0	0	1	1	1	1	1	$(b'_{-2})_0 \vee b(a \oplus x)(a_{-1} \oplus x_{-1}) \vee ax\bar{a}_{-1}x_{-1}$	$axa_{-1}\bar{x}_{-1} \vee a_{-1}x_{-1}$	
*	0	1	0	0	0	0	1			
*	0	1	0	0	0	1	1			
*	0	1	0	0	1	0	1			
*	0	1	0	0	1	1	1			
*	0	1	0	1	0	0	1			
*	0	1	0	1	0	1	1			
*	0	1	0	1	1	0	1			
*	0	1	0	1	1	1	1			
*	0	1	1	0	0	0	1			
*	0	1	1	0	0	1	1			
14	0	1	1	0	1	0	1	$(b'_{-2})_0 \vee b(a \oplus x)\bar{a}_{-1}x_{-1} \vee ax(a_{-1} \oplus x_{-1})$	$b(a \oplus x)a_{-1}\bar{x}_{-1} \vee a_{-1}x_{-1}$	
15	0	1	1	0	1	1	1	$(b'_{-2})_0 \vee b(a \oplus x)\bar{a}_{-1}x_{-1} \vee b\bar{a}xa_{-1}\bar{x}_{-1} \vee ax(a_{-1} \oplus x_{-1})$	$ba\bar{x}a_{-1}\bar{x}_{-1} \vee a_{-1}x_{-1}$	
*	0	1	1	1	0	0	1			$tax(a_{-1} \oplus x_{-1})$
*	0	1	1	1	0	1	1			
16	0	1	1	1	1	0	1	$(b'_{-2})_0 \vee b(a \oplus x)\bar{a}_{-1}x_{-1} \vee ba\bar{x}a_{-1}\bar{x}_{-1} \vee ax(a_{-1} \oplus x_{-1})$	$b\bar{a}xa_{-1}\bar{x}_{-1} \vee a_{-1}x_{-1}$	
17	0	1	1	1	1	1	1	$(b'_{-2})_0 \vee b(a \oplus x)(a_{-1} \oplus x_{-1}) \vee ax(a_{-1} \oplus x_{-1})$	$a_{-1}x_{-1}$	
18	1	1 or 0	1	1	1	1	1	$(b'_{-2})_0 \vee b(a \oplus x)(a_{-1} \oplus x_{-1}) \vee ax(a_{-1} \oplus x_{-1}) \vee a_{-1}x_{-1}$	0	

*These cases violate the condition (23).

**In this case $b'_{-2}a' = (t \oplus a \oplus x)a_{-1}x_{-1} \vee tax(a_{-1} \oplus x_{-1})$, and $b'_{-2}a'_{-1}$ is not void either;

$$b'_{-2}a_{-1} = [t(a \oplus x) \vee ax]a_{-1}x_{-1}.$$

because this relation will possibly contribute to simplifying the structure of the circuits associated with the adder such as the high speed carry assimilator, the division predictor, etc.

Among them let us examine the structure numbered 9, which seems to be the simplest, in designing the detailed logical circuit with the use of the emitter-follower diode logic of the New Illinois Computer basic circuitry, in which the fan-out both from the emitter-follower and from the AND-gate is restricted to two and the number of cascaded stages is limited to six.

The basic Boolean equations are as follows:

$$t = p' \vee b$$

$$a' = t \oplus a \oplus x$$

$$u_{-1} = t\ (\oplus x) \vee a\ x$$

$$a'_{-1} = u_{-1} \oplus a_{-1} \oplus x_{-1}$$

$$b'_{-2} = t\ (a \oplus x)\ (a_{-1} \oplus x_{-1})$$

$$p'_{-2} = a\ x\ (a_{-1} \oplus x_{-1}) \vee a_{-1}\ x_{-1}$$

Since NOT operation cannot be done by the emitter-follower and diode logic, it is necessary to provide $\overline{p}'_{-2}$ besides p'_{-2}. It turns out that it is more economical to generate $\overline{p}'_{-2}$ and t_{-2} which is $p'_{-2} \vee b_{-2}$, assuming in turn $\overline{p}$ and t are propagated from the previous lower significant stage.

$$\beta = a\ \overline{x} \vee \overline{a}\ x$$

$$\overline{\beta} = a\ x \vee \overline{a}\ \overline{x}$$

$$a' = t\overline{\beta} \vee \overline{p}'\ \overline{b}\ \beta$$

$$\gamma = a\ x$$

$$\overline{\gamma} = \overline{a} \vee \overline{x} = \overline{a}\ \overline{x} \vee \overline{a}\ x \vee a\ \overline{x}$$

$$q = t\beta$$

$$\overline{q} = \overline{t} \vee \overline{\beta} = \overline{p}'\ \overline{b} \vee t\overline{\beta}$$

$$\beta_{-1} = \beta^{*}_{-1} = a_{-1}\ x_{-1} \vee \overline{a}_{-1}\ x_{-1}$$

$$\overline{\beta}_{-1} = a_{-1}\ x_{-1} \vee \overline{a}_{-1}\ \overline{x}_{-1}$$

$$a'_{-1} = \gamma\ \overline{\beta}_{-1} \vee q\overline{\beta}_{-1} \vee \overline{\gamma}\ \overline{q}\ \beta_{-1}$$

$$p'_{-2} = \overline{a_{-1}\, x_{-1}} \quad [\overline{a\, x} \vee \overline{a_{-1} \oplus x_{-1}}]$$

$$= (\bar{a}_{-1}\, \bar{x}_{-1} \vee \beta_{-1})\, (\bar{\gamma} \vee \bar{\beta}_{-1})$$

$$= \bar{\gamma}\, \beta_{-1} \vee \bar{\gamma}\, a_{-1}\, x_{-1} \vee a_{-1}\, x_{-1}$$

$$= \bar{\gamma}\, \beta_{-1} \vee a_{-1}\, x_{-1}$$

$$b_{-2} = q\beta_{-1}^{*}$$

We need both β_{-1} and β_{-1}^{*} of the same logical function because of the limited fan-out. It turns out that 21 emitter followers and 95 diodes are necessary as shown in Figure 2. These are slightly more than 19 transistors and 85 diodes in G. A. Metze's circuit(1) for structure No. 17 in Table 1.

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(1) Supplement for File No. 273 of Digital Computer Laboratory

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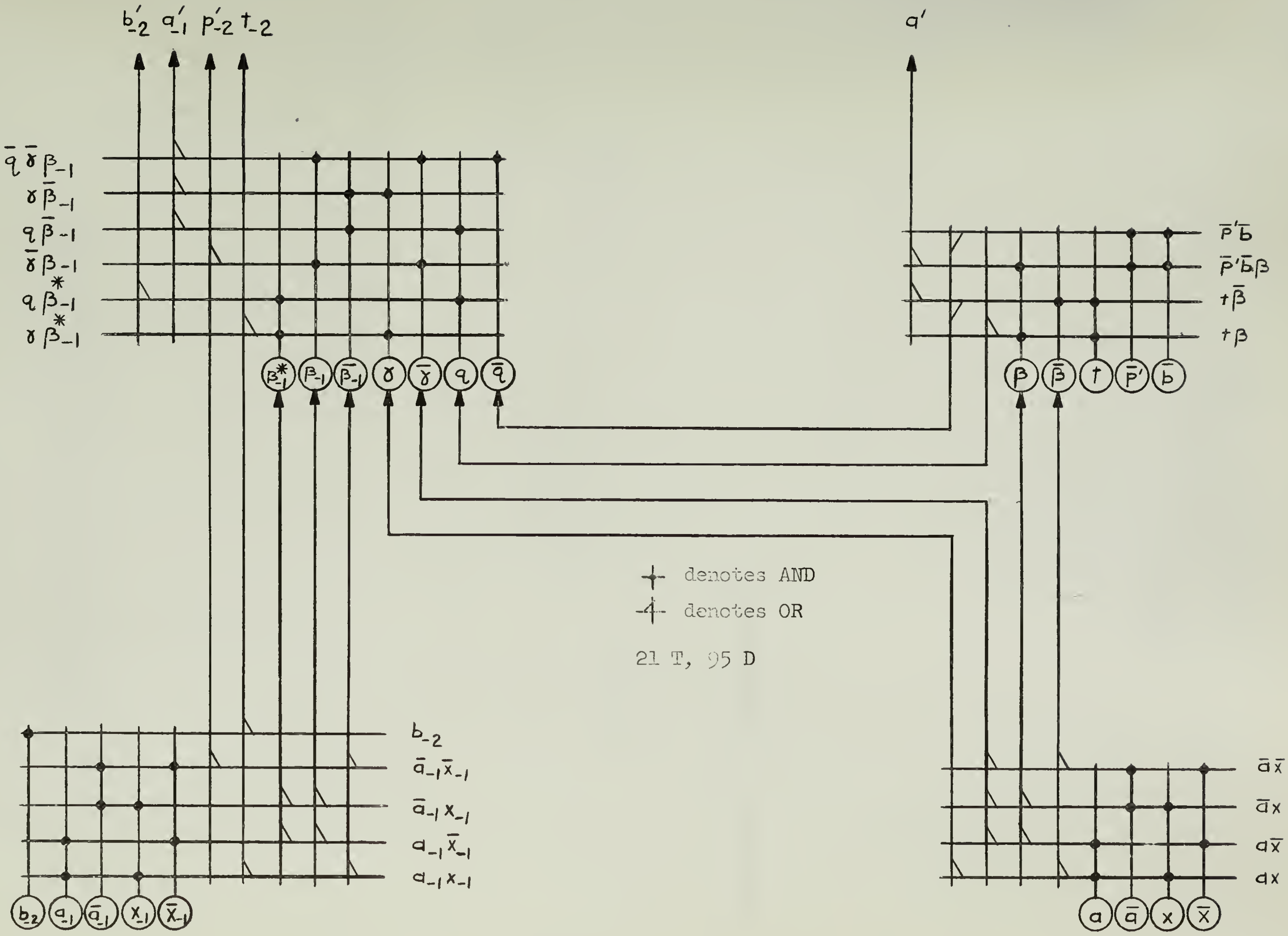

Figure 2